Symbolic Transformations of Dynamic Optimization Problems

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Dynamic optimization problems occur in various fields and applications. These include parameter estimation and optimal control. Optimization problems involving differential-algebraic equation (DAE) systems are traditionally solved while retaining the semi-explicit or implicit form of the DAE. We instead consider symbolically transforming the DAE into an ordinary differential equation (ODE) prior to solving the optimization problem using a collocation method. We present a method for achieving this that treats DAE-constrained optimization problems. The method is based on techniques commonly used in Modelica tools for simulation of DAE systems. These techniques involve causalization of DAE systems and transformation to block-lower triangular form of the equations.

The method has been implemented in the JModelica.org open-source Modelica platform. The implementation is evaluated on two industrially relevant benchmark problems. The first problem is a minimum-time problem, where we seek the time-optimal maneuver for an automobile in a hairpin turn, see Figure 1. The second problem is to optimize the warm startup of a combined-cycle power plant, whose model diagram is displayed in Figure 2.

Figure 1: An example of a hairpin turn. Photo courtesy of RallySportLive.

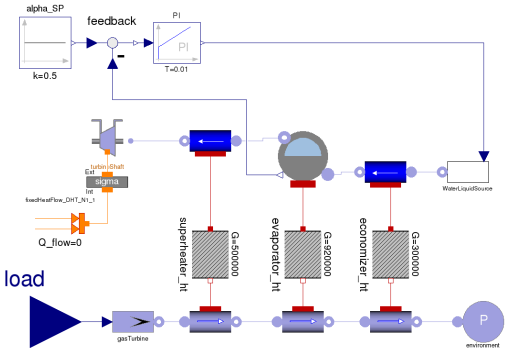

Figure 2: Power plant model diagram

The problems are solved using both the traditional DAE formulation and the proposed ODE formulation. The performance of the two approaches is compared. The ODE formulation is shown to have between two and four times shorter online execution time for the considered cases. We also discuss benefits and drawbacks of the two approaches.