Efficient Numerical Integration of Dynamical Systems based on Structural-Algebraic Regularization avoiding State Selection

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Differential-algebraic equations (DAEs) naturally arise in the modeling of dynamical processes, in particular using MODELICA as modeling language. In general, the model equations can be of higher index, i.e., they can contain *hidden constraints*, which lead to instabilities and order reductions in the numerical integration. Therefore, a regularization or remodeling of the model equations is required. One way to obtain the required information on the hidden constraints is a structural analysis based on the sparsity pattern of the system. For the determination of a regular index-reduced system formulation then, usually, a crucial step is the *state selection* that is required in order to introduce new algebraic variables for the selected differential components of the DAE system. However, the choice of states that are selected can change during the numerical integration.

In this paper, we present a new regularization approach for the remodeling of quasi-linear DAEs that uses the information obtained from the structural analysis, in particular from the Signature Method [2], to construct a regularized overdetermined system formulation. The structural analysis provides the required information that allows to extract all hidden constraints. Adding these hidden constraints to the system leads to an overdetermined system formulation that has the great advantage that all constraints are stated in explicit form and no further analytical manipulations for the determination of a square and uniquely solvable system have to be applied. This overdetermined system can then be solved using a specially adapted numerical integrator that ensures that all constraints are satisfied during the numerical integration. Thus, the state selection is performed within the numerical integrator during runtime of the simulation. A further advantage of an overdetermined regularization is the possibility to add solution invariants, e.g., conservation laws, to the constraints, which often stabilizes the numerical integration.

Currently, no MODELICA simulation framework is able to handle overdetermined system formulations. Therefore, a prototype MODELICA framework `MPSSim` is presented that uses the translator `MO2FOR` [1] to translate an overdetermined system model provided in MODELICA into FORTRAN source code which can then be integrated using the software package `QUALIDAES` that is suited for the direct numerical integration of regularized overdetermined model equations. A comparison of `MPSSim` with simulation tools like `MapleSim`, `Dymola` and `OpenModelica` shows its promising performance.

References

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