Efficient Monte Carlo simulation of stochastic hybrid systems

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This paper proposes an efficient approach to model stochastic hybrid systems and to implement Monte Carlo simulation for such models, thus allowing the calculation of various probabilistic indicators: reliability, availability, average production, life cycle cost etc. Stochastic hybrid systems can be considered, most of the time, as Piecewise Deterministic Markov Processes (PDMP). Although PDMP have been long ago formalized and studied from a theoretical point of view by Davis (*Davis 1993*), they are still difficult to use in real applications. The solution proposed here relies on a novel method to handle the case when the hazard rate of a transition λ depends on continuous variables of the system model, the use of an extension of Modelica 3.3 and on Monte Carlo simulation. We illustrate the approach with a simple example: a heating system subject to failures, for which we give the details of the modeling and some calculation results. We compare our ideas to other approaches reported in the literature.

Typical simulation results for the example are shown in the figure below. To perform a Monte Carlo simulation to estimate, for example, the mean temperature as a function of time, it is necessary to generate a large number of trajectories using different initial seeds for every simulation. This task can be performed in Dymola by using appropriate script functions (that are based on the algorithmic part of the Modelica language). A special Modelica/Dymola script has been implemented for this case to run the simulations and store the desired fractiles. In Figure 1, the mean value of the room temperature is shown, as well as the 1% and 99% fractiles at each time point respectively. 10 000 simulations were performed with 500 output points per simulation. On a notebook, these simulations took 25s.

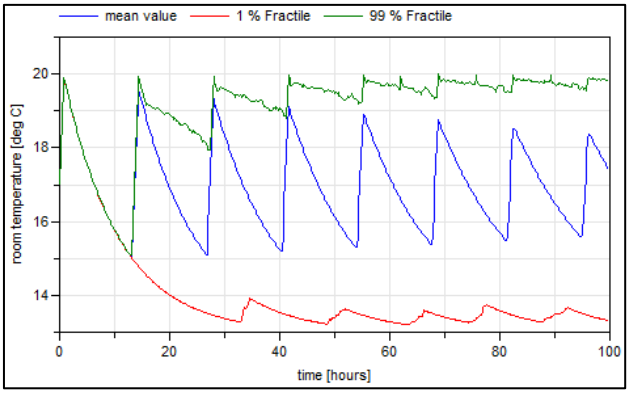

Figure 1: Statistics about room temperature (based on heating system subject to failures) obtained from 10000 trajectories