A Generalized Power-Based Modelica Library with Application to an Industrial Hydraulic Plant

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Abstract

This contribution presents a Modelica implementation of the generalized power-based modeling approach conforming to the bond graph methodology. The corresponding developed library BondGraph is discussed in detail. It allows graphical modeling according to the bond graph formalism, and contains common bond graph elements, as well as specific nonlinear elements, especially related to hydraulic effects. Furthermore, several composed models are provided, such as switching valves, pipes, cylinders, etc. A combination with blocks of the Modelica Standard Library is possible. The application of BondGraph to an industrial plant is described to demonstrate its capabilities.

Keywords: *power-based model, bond graph, library, Modelica, hydraulic*

1 Introduction

Since general purpose simulation environments enhance their capabilities, modelers attempt to take advantage of the offered possibilities completely. Hence, models become more comprehensive and cover the multidisciplinarity of the considered systems. Therefore, domain specific modeling libraries are developed and offer possibilities to model the involved different physical effects separately as well as to interconnect them. The generalized power-based approach is an alternative efficient modeling formalism. Based on the generalized power definition, unified modeling elements are conceptualized applicable to the direct multidisciplinary modeling of complex systems. Consequently, the modeling procedure consists of the characterization of domain specific processes corresponding to their unified complements and interconnecting these complements to the model of complete system according to its structure. For further advantages of generalized power-based modeling, see e.g. [1] and [2]. The Modelica language is particularly suitable for the development and representation of power-based models, primly because of the possibility of the acausal multiple signal connections definition.

The bond graph (BG) formalism provides a multidisciplinary, generalized power-based approach to the modeling and also graphical model representation of dynamic systems (cf. [3]). By the object-oriented nature, BG possess the according advantages. The graphical system representation by a BG can be translated automatically to a system of differential-algebraic equations, wherefore the implementation of the BG approach with an appropriate modeling language is necessary.

The Modelica language and the BG formalism are partly closely related modeling methodologies. Hence, an implementation of the BG formalism in Modelica is considered in this contribution. The proposed implementation attempts to take advantages of both modeling concepts. The developed open source library BondGraph is recommended to be used with Dymola 7 or later versions as this software supports completely included model definitions.

The introduced BondGraph library is available online at the official Modelica web page [4]. It contains common standard BG elements, as well as specific nonlinear elements. The included nonlinear models were designed especially for the modeling of hydraulic networks. For these, attention has been paid to their numerically stable computation. Further components provided by BondGraph include composed models of technical units, e.g. valves, and blocks for signal generation, e.g. to control valves.

A short introduction to the BG formalism is given in the Section 2. The implementation of a number of representative elements in the BondGraph library is

discussed in Section 3. In Section 3.1, the implementation of the standard BG elements, and in Section 3.2, the implementation of the hydraulic elements are presented. Section 4 introduces an industrial plant and the corresponding model developed using the BondGraph library. Several examples are provided to illustrate the described issues. A brief summary is given in Section 5. The used formula symbols are summarized in the Appendix.

2 Bond Graphs

Since its development, the BG approach has become a well-known technique for object-oriented graphical modeling (cf. [3]). This fact is based on the established definition of generalized power (cf. [5]) as the product of a conceptional effort e and a flow f variable:

$$P = f \cdot e. \tag{1}$$

This generalized specification provides the feasibility of multidisciplinary modeling with unified elements. The only prerequisite for this obviously advantageous modeling approach is an appropriate assignment of power variables i.e. effort and flow for considered domains.

For example, in the field of isothermal hydraulics, conventionally the pressure and the volumetric flow rate are assigned as the effort and the flow variable, respectively. A list of corresponding possible assignments for different energy domains is shown in Table 1.

Table 1: A selection of domains and corresponding conventional variable assignments in the bond graph formalism

Domain	Flow	Effort
hydraulic	volume flow rate	pressure
translational	velocity	force
rotational	angular velocity	torque
electrical	current	voltage
thermal	entropy flow rate	temperature

2.1 Bond Graph Elements

The basic set of BG elements consists of an effort source, a flow source, a capacitance, an inductance, a resistance, a transformer, and a gyrator, which are given with corresponding constitutive equations in Table 2.

Table 2: Basic bond graph elements

Name	Element	Constitutive Equation
effort source	Se	$e = e_s$
flow source	Sf	$f = f_s$
capacitance	C	$e = \frac{1}{c} \int_{t_0}^{t} f(t^*)dt^* + e(t_0)$
inductance	I	$f = \frac{1}{i} \int_{t_0}^{t} e(t^*)dt^* + f(t_0)$
resistance	R	$e = r \cdot f$
transformer	TF	$e_1 = r_{tf} \cdot e_2,\ f_1 = \frac{1}{r_{tf}} \cdot f_2$
gyrator	GY	$e_1 = r_{tf} \cdot f_2,\ f_1 = \frac{1}{r_{tf}} \cdot e_2$

A significant extension of the modeling opportunities is offered by modulated elements. These are generalizations of linear elements where the proportionality coefficients are given by an external signal.

Two further multi-port elements referred to as junctions are defined in the bond graph methodology. These may represent conservation or equilibrium laws but also design constraints among the variables of the elements connected. In the graphical representation, the two power variables are carried by one eponymous bond connecting junctions and other BG elements, that is usually drawn as a half arrow. The flows of all elements connected to a 1-Junction are equal, whereas the sum of efforts carried by all incoming bonds equals the sum of efforts carried by all outgoing bonds. Again a 0-Junction is a dual element corresponding to the 1-Junction, hence for this the reversed relations of effort and flow are valid. For example, related to the hydraulic domain, all elements connected to a 1-Junction have the same flow as being connected in series, and all elements connected to a 0-Junction have the same effort as being connected in parallel.

Formally, a 1-Junction is described by the equations

$$\sum_{k=1}^{n} s_k \cdot e_k = 0,\ s_k \in \{+1, -1\}, \tag{2}$$

$$f_1 = f_2 = \cdots = f_n, \tag{3}$$

where n equals the number of elements connected to the junction. In the same manner, a 0-Junction is described by

$$\sum_{k=1}^{n} s_k \cdot f_k = 0,\ s_k \in \{+1, -1\}, \tag{4}$$

$$e_1 = e_2 = \cdots = e_n. \tag{5}$$

By convention, $s_k = +1$ for all incoming bonds and $s_k = -1$ for all outgoing bonds. Hence, the contributions to the constitutive equation of a junction are determined by the directions of the bonds in the graphical representation. Nevertheless, as can be seen by (2) and (4), formally, this is rather a property of the junction itself than of the connection. This fact is considered by the proposed implementation of the BG formalism in Modelica.

According to a specific domain, for each standard BG element a physical interpretation can be given. Considering a combination of linear elements, modulated elements, and sensors observing power variables, it is possible to describe a wide range of nonlinear models by case-related definitions of the external signal. Nevertheless, the definition of application-specific, and, where appropriate, nonlinear elements may lead to a convenient modeling process and ensures the clarity of the obtained models.

2.2 Causality

Within the BG formalism, bond connections are representations of information exchange between submodels. As two power variables are associated with each bond, two equations are obliged for their calculation and thus each end of the bond is responsible for one of them. Graphically, the flow determining end is marked by a short stroke. For example, energy storage elements are preferred to have integral causality, thus a capacitance preferably computes effort and an inductance preferably computes flow, whereas sources obviously determine their output variables. Several rules exist for causality assignment, e.g. listed in [3].

The graphical assignment of causality is useful if the equations describing the system behavior are directly deduced from the graphical BG representation. In some modeling tools as for example 20-sim (cf. [6]), the causality is fixed during the modeling procedure. However, it is not required to assign the causality during the modeling process using Modelica, as this is a subtask of the model compilation process, which is performed by suitable matching algorithms. For large models and generally for practical modeling, the manually or a priori fixed causality assignment is less reasonable. Therefore, manual causality assignment is not integrated in the presented BG implementation.

2.3 Example

An example of a simple physical system and its BG representation are shown in Fig. 1. The considered system is a hydraulic cylinder driven by enforced pressures at hydraulic ports assigned with PT_1 and PT_2. On the mechanical side, a load is executed represented by a force at the corresponding port PT_3. In this simple representation, basic BG elements are utilized exclusively, which may be less suitable for modeling with practical accuracy requirements.
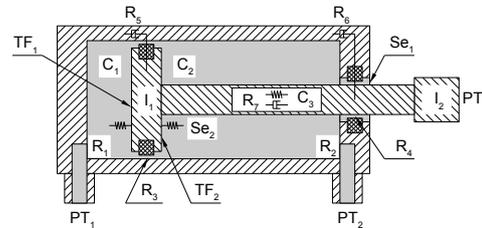
(a) Iconic model of a hydraulic cylinder
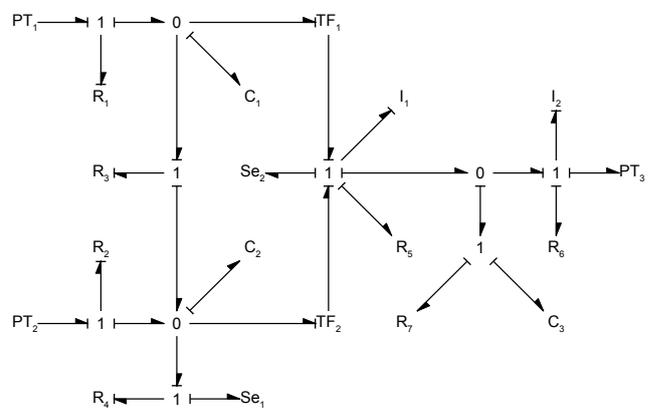
(b) Classical bond graph representation

Figure 1: Simple bond graph example

3 BondGraph Library

3.1 Standard Bond Graph Elements

The set of standard bond graph elements is comprised of basic BG elements, their modulated complements, and junctions. Thus, these are included in the BondGraph library. Therefore, the corresponding constitutive equations and element interfaces are implemented in Modelica. The power variables are made accessible by acausal connectors, whereas for signals, input-output connectors are defined compatible to blocks of the Modelica Standard Library (MSL). For the energy storage elements, initial conditions are defined locally applying start attributes for differentiated variables. The element parameters are implemented according to the BG methodology. Extending this set, sensors are modeled as simple two port elements that do not affect the power variables. Via a

signal output interface, these elements provide power variables or their product i.e. power, corresponding time derivatives, or integrals. Again, the compatibility with blocks of the MSL is assured.

3.1.1 Bonds

According to the bond graph methodology, elements are interconnected by bonds. In contrast to the bonds, Modelica connections do not provide attributes for variable sign and for causality assignment, and their graphical representation. Therefore, if the bonds should be implemented closely to the original methodology, they would have to be objects of a class containing required properties. This approach would make the modeling procedure significantly more cumbersome.

The BondGraph library uses standard Modelica connections for the element interconnection introducing the connect equation directly or graphically. The graphical modeling technique is recommended here, as the BG formalism is a graphical modeling approach. Hence, an alternative method is used to determine the signs of power variables at a junction (cf. (2) and (4)). Therefore, junctions are equipped with a positive and a negative multiple port. These are indicated by a blue and a red circle, respectively. All elements connected to the positive port are considered with a positive sign ($s_k = +1$) and all elements connected to the negative port are considered with a negative sign ($s_k = -1$) in the sum of efforts for a 1-Junction, or in the sum of flows for a 0-Junction. In terms of the standard BG formalism, all incoming bonds are connected to the positive port and all outgoing bonds are connected to the negative port. This yields a clear graphical representation in compliance with (2) and (4) as discussed in Section 2.1.

3.1.2 Causality Assignment

As it has been discussed in Section 2.2, the causality assignment is not required by using Modelica. Furthermore, restrictions on algorithmic matching procedures may lead to less efficient model equations resolution. Hence, any regulations of this kind are avoided in the BondGraph library. Nevertheless, a graphical indication of the BG formalism specific causality assignment can be implemented in the animation layer extending the models of the released library by appropriate functionality. In this way, the standard BG causality assignment method might be visualized without eventual affecting of the equation resolution procedure.

3.1.3 BondGraph Example

The implementation of the example presented in Section 2.3 is shown in Fig. 2, now using the BondGraph library in Dymola. Here, the pressures and force imposed at the hydraulic and mechanical ports, respectively, are implemented utilizing effort sources and look-up tables from the MSL.

Figure 2: Simple bond graph example implemented using BondGraph

3.2 Hydraulic Elements and Effects

Hydraulic processes are subprocesses of fluid-mechanical power exchange. For the modeled processes, this is considered transient and spatially one-dimensional along a main flow direction. Models of the following hydraulic effects are included in the BondGraph library: capacitive energy storage (*HC* and *MHC*), inductive energy storage (*HI*), dissipation (*HR*, *HRL*, *HRT*, and *MHRT*), and processes with source characteristics (*HSe acc* and *HSe ind*). In the hydraulic domain, the power variables *e* effort and *f* flow are pressure and volume flow, respectively.

3.2.1 Working Fluid Properties

For the modeling of these hydraulic processes, the properties of the working fluid are of essential interest, since these may vary significantly within the working range. The *hprop* model of the BondGraph library provides a computation of the hydraulic fluid properties density and viscosity depending on the fluid temperature and the absolute pressure. By default, it is parameterized concerning the working fluid HLP ISO VG 32 as specified by the German industry norm DIN 54524, part 2. The introduced model is valid for the temperature range of $[288, 363]$ K and the feasible pressure lies in a wide range of $[1, 10^8]$ Pa. The properties computed by the *hprop* model play an important role for

other hydraulic effects and therefore it is contained in the models of hydraulic processes.

For each process, a decisive pressure is defined by the arithmetic mean of the pressures before and after the considered process. This definition assures the independence of the flow direction and is of sufficient accuracy in most cases. Different to the standard bond graph convention, hydraulic processes are implemented as two-port elements in the BondGraph library where the mean values of the pressures at the ports, and also the fluid properties as determined by *hprop*, are automatically considered.

The dependence of fluid properties upon other conditions besides pressure and temperature is not modeled. The working fluid is assumed to be a homogeneous mixture of hydraulic oil and air, whereby the relative mass parts in the mixture are assumed to be time and state invariant.

Density. The density of the hydraulic oil is described by the equation

$$\rho_{oil} = \rho_{oil,ref} \cdot \exp\left(\kappa_{oil} \cdot (p - p_{ref}) - \gamma_{oil} \cdot (T - T_{ref})\right), \tag{6}$$

considering the compressibility and thermal expansion of the oil by the parameters κ_{oil} and γ_{oil}, respectively, whereby the density of air is modeled by the ideal gas law

$$\rho_{air} = \rho_{air,ref} \cdot \frac{p}{p_{ref}} \cdot \frac{T_{ref}}{T}. \tag{7}$$

These densities are then used to obtain the the density of the mixture by

$$\rho = \left(\frac{\mu_{oil}}{\rho_{oil}} + \frac{\mu_{air}}{\rho_{air}}\right)^{-1}, \tag{8}$$

which may be parameterized by the relative mass parts of the hydraulic oil and air.

It is also possible to obtain the compensated compressibility modulus of the working fluid from this model (cf. [7]). Thus it was verified with the conversed available data for the compensated compressibility modulus.

Viscosity. The viscosity of the hydraulic oil is modeled by the Roelands relation (cf. [8])

$$\eta_{oil} = \eta_{oil,ref} \cdot \exp\left(\ln\left(\frac{\eta_{oil,ref}}{6.315 \cdot 10^{-5}\text{Pa} \cdot \text{s}}\right) \cdot \psi\right), \tag{9}$$

with

$$\psi = -1 + \left(1 + \frac{p - p_{ref}}{1.96 \cdot 10^8 \text{Pa}}\right)^\zeta \cdot \left(\frac{T - 138\text{K}}{T_{ref} - 138\text{K}}\right)^\xi. \tag{10}$$

The viscosity of air is described by the own approximation

$$\eta_{air} = \eta_{air,ref} \cdot \frac{\theta(p, T)}{\theta(p_{ref}, T_{ref})}, \tag{11}$$

with

$$\theta(p, T) = p^{0.01} \cdot T^{0.75} + 0.132 \cdot p \cdot T^{-2}. \tag{12}$$

Data used for the approximation is available in [9].

For the calculation of the viscosity of the mixture, relative volume parts of the components are obtained by

$$\phi_{oil} = \frac{\frac{\mu_{oil}}{\rho_{oil}}}{\frac{\mu_{oil}}{\rho_{oil}} + \frac{\mu_{air}}{\rho_{air}}} \tag{13}$$

and

$$\phi_{air} = \frac{\frac{\mu_{air}}{\rho_{air}}}{\frac{\mu_{oil}}{\rho_{oil}} + \frac{\mu_{air}}{\rho_{air}}}. \tag{14}$$

With reference to it's physical characteristic, the viscosity of the homogeneous mixture of hydraulic oil and air is then described by

$$\eta = \frac{\eta_{oil} \cdot \phi_{oil}^{2/3} + \eta_{air} \cdot \phi_{air}^{2/3}}{\phi_{oil}^{2/3} + \phi_{air}^{2/3}}. \tag{15}$$

3.2.2 Hydraulic Capacitance

The capacitive storage capability has a significant effect on the dynamic behavior of hydraulic systems. This property results from the elasticity of the considered hydraulic component and consequently from the pressure dependent variability of the enclosed fluid volume in the component. At low pressure, the compressibility of the working fluid has a considerable effect on the capacitive storage capability. Both effects are considered by the hydraulic capacitance model *HC* and mathematically described by

$$f = \frac{1}{\rho} \cdot \frac{d}{dt}(\rho \cdot v), \tag{16}$$

with

$$v = v_0 \cdot \exp(c_v \cdot (e - e_0)), \tag{17}$$

where e is the pressure, e_0 is the initial pressure, and f is the volume flow difference taken by the component from the main volume flow.

Whereas the initial volume v_0 is a parameter in the *HC* model, it can be adjusted by an external signal in the *MHC* model. This extends the modeling opportunities and e.g. allows the consideration of components with moving parts as for instance cylinders.

3.2.3 Hydraulic Inductance

From a power-based point of view, the inductive energy storage is the dual process to the capacitive storage and it mainly results from the inertia of the working fluid. The relation between the power variables in the inductive storage element is obtained from the principle of linear momentum applied on the enclosed working fluid in the component by

$$e = \frac{d}{dt}\left(\frac{l \cdot \rho}{A} \cdot f\right), \qquad (18)$$

where e is the pressure difference across the component.

3.2.4 Hydraulic Dissipation

Dissipative hydraulic processes are highly nonlinear and change their characteristic dependent on the transient flow conditions (cf. [10]). As the hydraulic resistance is described using different models according to the particular effect, several elements are provided by the BondGraph library: strait pipe resistance (*HR*), resistance of a pipe fitting with laminar flow (*HRL*), resistance of a pipe fitting with turbulent flow (*HRT*), and switchable hydraulic resistances (*MHRT*) are distinguished.

Strait pipe resistance. The dissipative resistance of strait pipes is extensively studied in the literature (cf. e.g. [11], [12], [13]). The model proposed in this contribution (given by the following set of equations (19)-(25)) is based on the Darcy equation (equation (19), cf. [14]). The pipe friction factor for the entire range of the Reynolds number is obtained with an explicit, continuous approximation of the pipe friction factor equation for laminar flow and of the Colebrook-White equation (cf. [15]).

As can be seen by Fig. 3, the resulting pipe friction factor function $\lambda(Re)$ (cf. equation (22)) is continuous as well as continuously differentiable, it does not diverge, and also does not include distinctions between different cases (if-then contructs), which yields stable computability. Due to the explicit form of equation (24), our approach also avoids the computationally expensive solution of the originally proposed implicit equation for turbulent flows.

$$e = \lambda \cdot \frac{l \cdot \rho}{2 \cdot d_h \cdot A^2} \cdot |f| \cdot f \qquad (19)$$

$$Re = \frac{d_h \cdot \rho \cdot |f|}{A \cdot \eta} \qquad (20)$$

$$Re_t = \frac{2 \cdot 6.9}{1 - \left(\frac{r_h}{3.7}\right)^{1.11}} \qquad (21)$$

$$\lambda = \frac{\lambda_l}{1 + \exp\left(\frac{Re - Re_{crit}}{0.228 \cdot Re_{range}}\right)} + \frac{\lambda_t}{1 + \exp\left(\frac{Re - Re_{crit}}{-0.228 \cdot Re_{range}}\right)} \qquad (22)$$

$$\lambda_l = \frac{64}{Re + Re_{min} \cdot \left(1 - \tanh\left(\frac{Re}{Re_{min}}\right)\right)} \qquad (23)$$

$$\lambda_t = \left(\frac{-1.8}{\ln(10)} \cdot \ln\left(\left(\frac{r_h}{3.7}\right)^{1.11} + \chi\right)\right)^{-2} \qquad (24)$$

$$\chi = \frac{6.9}{Re + Re_t \cdot \left(1 - \tanh\left(\frac{Re}{Re_t}\right)\right)} \qquad (25)$$

In this equation set, e is the pressure difference across the component. To obtain numerical stability using sigmoidal functions, the following parameters are utilized: Re_{crit} is the Reynolds number value for the laminar-turbulent flow transition, Re_{range} is the Reynolds number range for the laminar-turbulent flow transition and Re_{min} is the Reynolds number value for the zero-flow crossing.
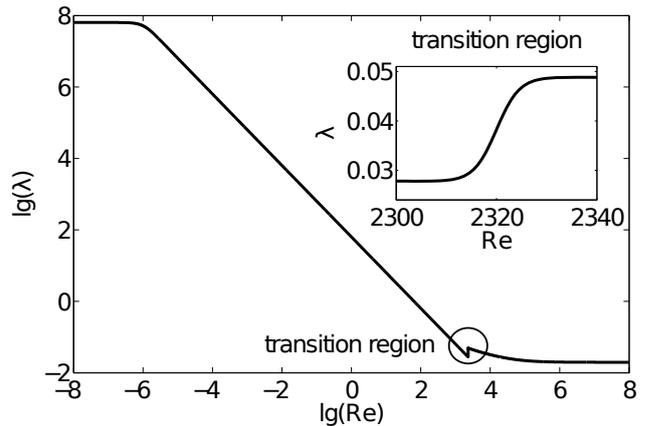
Figure 3: Proposed approximation of pipe friction factor λ according to equation (22). The inset shows the laminar-turbulent transition region in detail with typical values for Re_{crit} and Re_{range}.

Resistance of pipe fittings. The effect of dissipation due to hydraulic fittings is available by the models *HRL* and *HRT*. These may be used in case of hydraulic apertures, throttles, bendings, cross-section changes, and similar components. Resistances in these components (cf. [7]) can be described by taking their flow characteristic into account. In case of laminar flow, the viscosity of the fluid strongly affects the resistance which is expressed by the equation

$$e = \frac{e_{ref} \cdot \eta}{\eta_{ref} \cdot f_{ref}^\gamma} \cdot |f|^{\gamma - 1} \cdot f. \qquad (26)$$

Contrariwise, in case of turbulent flow, the density of the fluid has to be considered:

$$e = \frac{e_{ref} \cdot \rho}{\rho_{ref} \cdot f_{ref}^{\gamma}} \cdot |f|^{\gamma-1} \cdot f. \tag{27}$$

In both cases, e is the pressure drop across the component. For both laminar and turbulent resistances, often flow and pressure drop are not in linear relation to each other, but their relation may be described by a characteristic flow exponent γ, e.g. $\gamma = 2$ if the flow is proportional to the square root of the effort. However, it usually depends on the geometry of the fitting and may therefore be adjusted by a parameter of the model.

Switchable hydraulic resistance. Hydraulic valves may be interpreted as switchable resistances and thus be modeled by switchable hydraulic elements. The *MHRT* element represents a signal controlled hydraulic resistance with turbulent characteristic where an input signal allows to control the state of the hydraulic resistance. An input value of 1 corresponds to a complete opened valve, whereas a value of 0 corresponds to a completely closed valve. To generate appropriate control inputs, several blocks are also provided by the BondGraph library. In this way the physical multi-domain nature of valves may be taken into account: though valves affect the hydraulic resistance, they are controlled by mechanical systems which are actuated either manually or by further technical units, e.g. a solenoid actuators. Therefore, array types of power-based models can be efficiently used for the modeling of hydraulic valves (cf. [16]).

3.2.5 Processes with Source-Characteristic

Height change. In the fluid-mechanical power exchange processes, two subprocesses with effort-source-characteristic are often involved. One of them is the pressure change e due to fluid displacement in the earth gravitation field, therefore

$$e = \rho \cdot g \cdot d, \tag{28}$$

which is considered by the *MSe acc* model.

Cross section area change. The second subprocess is covered by the *MSe ind* element and models the pressure change e due to a cross sectional area change in hydraulic lines, thus

$$e = \frac{\rho}{2} \cdot |f|^2 \cdot \left(\frac{1}{A_1^2} - \frac{1}{A_2^2} \right). \tag{29}$$

3.2.6 Consideration of Nonlinearity

Causalized elements. As pointed out in Section 2.2, causality does not have to be assigned in general by using Modelica. Nevertheless, when using nonlinear elements e.g. *HR*, *HRT*, *HRL*, or *MHRT*, it is recommended that the designer observes whether the element has effort-out or flow-out causality in the actual model. Accordingly, the activation of an appropriate formulation of the element equations should be controlled by a parameter. In this way, unnecessary numerical inversion of the nonlinear element equations is avoidable. Hence, the solution time of the model can be shortened and the solution accuracy and stability can be increased.

For instance, in (30) and (31), both causalized forms of the turbulent resistance are given (cf. equation (27)).

$$e = \frac{e_{ref} \cdot \rho}{\rho_{ref} \cdot f_{ref}^{\gamma}} \cdot |f|^{\gamma-1} \cdot f \tag{30}$$

$$f = \left(\frac{e_{ref} \cdot \rho}{\rho_{ref} \cdot f_{ref}^{\gamma}} \right)^{\frac{-1}{\gamma}} \cdot |e|^{\frac{1}{\gamma}-1} \cdot e \tag{31}$$

In (32), a form corresponding to the linear resistance element is obtained. The state dependent resistance coefficient r is separated. Different formulations for the resistance coefficient are then used dependent on the par_{caus} parameter.

$$e = r \cdot f, \tag{32}$$

with

$$r = \begin{cases} \frac{e_{ref} \cdot \rho}{\rho_{ref} \cdot f_{ref}^{\gamma}} \cdot |f|^{\gamma-1} \cdot f & \text{if } par_{caus} = 1, \\ \left(\frac{e_{ref} \cdot \rho}{\rho_{ref} \cdot f_{ref}^{\gamma}} \right)^{\frac{1}{\gamma}} \cdot |e|^{1-\frac{1}{\gamma}} \cdot e & \text{if } par_{caus} = 2. \end{cases} \tag{33}$$

Variation of the proportionality factors. Corresponding reformulations and separations of the proportionality factors are often possible and convenient for nonlinear elements with other characteristics. As these coefficients in such reformulated relations are in general time and state variant, their absolute values can attain low values or zero. The same has to be considered for the reciprocal of the absolute value. In these cases, structural changes arise in the interconnected model equations and a numerical solution is either not possible or becomes unstable. For instance, this might result in a non-decaying oscillation of the power variables where the amplitude and particularly the frequency of the oscillation depend on the chosen

solver algorithm and solution tolerance. In this case, an effective and simple modification of the model is the limitation of the value of the proportionality coefficient. The mentioned numerically induced oscillations are avoidable completely by this model extension.

3.3 Comparison Remark to BondLib

BondLib is a former successful implementation attempt of the bond graph formalism in the Modelica language presented in [17] awarded as the best free Modelica library in the framework of the Modelica Conference 2005. BondGraph is a conceptually different modeling library, intended rather for pragmatic practical modeling using bond graph methodology then for educational purpose. In contrast to BondLib, BondGraph does not include elements closely corresponding to bonds. Instead, Modelica standard connections are utilized for the interconnection of BG elements. Consequently, the assignment of the variable signs is implemented as a property of junctions instead of using the directional property of bonds. In this way, BondGraph utilizes Modelica more efficiently slightly differing from the graphical representation defined in the BG methodology. Since bonds as elements are excluded, in comparison to a model set up with BondLib, an equivalent model set up with BondGraph involves significantly less elements. Furthermore, the graphical clarity for large models can be maintained more straightforward. The junction implementation in BondGraph permits connection of elements of unlimited number, whereas in the BondLib, junctions with different fixed numbers of ports are available. Hence, model modifications and expansions are performed more efficiently utilizing BondGraph, since originally used junctions may be retained. From this point of view, the BondGraph junction implementation is closer to the BG methodology. The BondLib library offers bonds with fixed causality. As described before, BondGraph avoids any regulations of this kind well motivated following more closely the Modelica approach. Concluding this short comparison, we suggest BondGraph for practical modeling, whereas we recognize that BondLib offers several properties very important for bond graph beginners and educational purpose.

4 Industrial Hydraulic Plant

The discussed library is utilized for the modeling of an industrial hydraulic plant. The considered plant is a riveting system with a hydraulic power transmission. The system can be structured into the following principal units: a hydraulic power supply unit containing an inductance motor and a pump, a valve block, a hydraulic-mechanical actuator, hydraulic pipes interconnecting these units, and a sequence control. Hence, the model to be developed should cover a comprehensive hydraulic network. Figure 4 shows the model top layer.
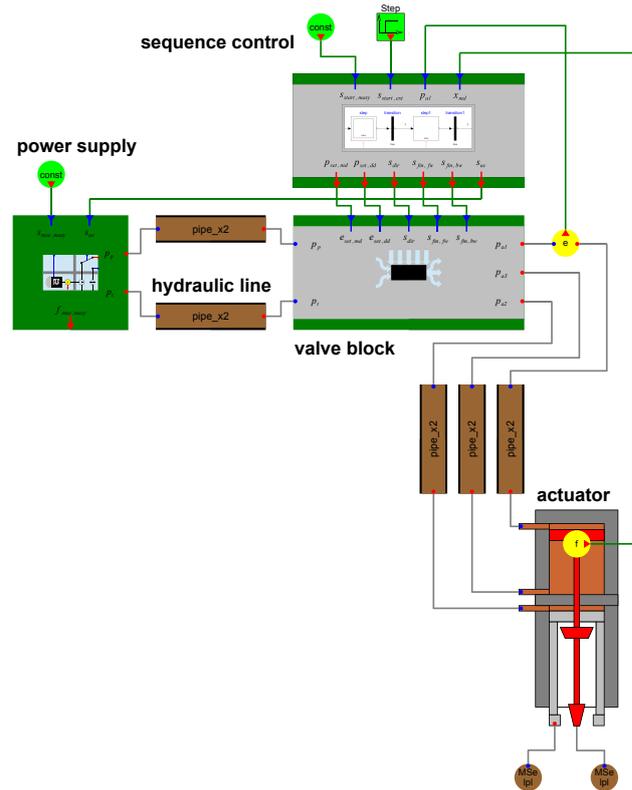
Figure 4: Model of the considered riveting system

The submodels representing principal units are modeled using the BondGraph library and the MSL. A plain example for this model set-up is the model of a hydraulic pipe consisting of hydraulic elements exclusively. A series of hydraulic dissipative and inductive elements separated by hydraulic capacitive elements may be utilized for the description of the pipe behavior. The series starts and ends with turbulent resistances representing resistances of the fittings. The inner part is the repeating of series consisting of a straight pipe resistance and a hydraulic inductance again separated by capacitive elements. This repeating incorporates spatial discretization of the pipe along the main flow direction. Consequently it should be undertaken an appropriate number of times to cover relevant system eigenvalues by the model. An example of pipe
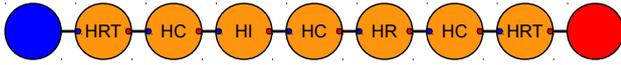
Figure 5: Model of the hydraulic pipe

model with one inductive storage is given in Fig. 5.

The model of the complete plant is a higher index differential-algebraic system consisting of 4923 equation with 106 continuous time states. The operation cycle of the plant of 1.7 s is simulated. Figure 6 shows the normalized displacement x of the actuator effector, the mechanical energy E supplied by the inductance motor in the power supply unit, and the pressure p in the hydraulic circuit.
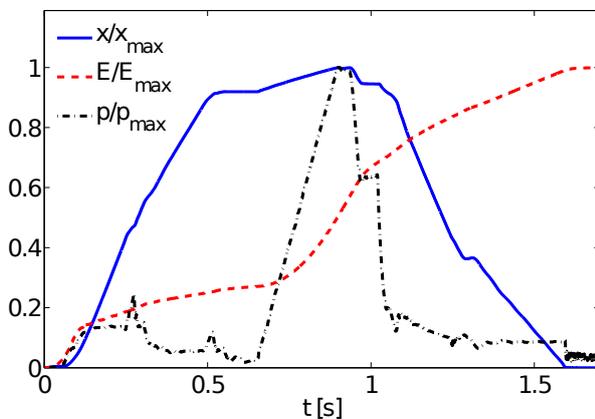
Figure 6: Simulation results. Plotted are the displacement x of the actuator, the mechanical energy E of the inductance motor, and the pressure p supplied by the pump.

The model parameters are identified by an optimization approach with reference to measurements obtained at a demonstrator plant. Hence, the model is verified against comprehensive experimental data. Furthermore, simulation based optimization of the plant is succeeded with the objective function defined as the consumed energy for an operation cycle (cf. [18]).

5 Conclusion

The bond graph methodology is a generalized power-based modeling approach. Hence, it is particularly advantageous for modeling of multidisciplinary systems. Therefore, BondGraph, the Modelica implementation of this approach enables applicants to take advantage of both, the flexible Modelica language and the well structured generalized power-based bond graph formalism. Furthermore, utilizing Modelica as the im-

plementation language, efficient general purpose modeling and simulation environments are made available for the bond graph applicants. Besides the development of BondGraph, a practical application of the library is conducted. Thereby, an industrial hydraulic plant is modeled, simulated, and optimized utilizing the developed modeling library.

Acknowledgments

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Appendix

List of Formula Symbols

A	cross section area of enclosed fluid volume
A_1	cross section area before change
A_2	cross section area behind change
c	capacitance coefficient
c_v	volumetric compliance factor
d	height difference
d_h	hydraulic diameter
E_{mech}	mechanical energy
$E_{mech,max}$	maximal mechanical energy value
e	effort
e_0	initial effort
e_k	effort at port k
e_{ref}	reference effort
e_s	source effort

f	flow
f_0	initial flow
f_k	flow at port k
f_{ref}	reference flow
f_s	source flow
g	gravitational acceleration
i	inductance coefficient
k	port counter
l	extend of enclosed fluid volume
n	port number
P	power
p	pressure
p_{ref}	reference pressure
par_{caus}	parameter for causalization
Re	Reynolds number
Re_{crit}	Reynolds number for laminar-turbulent transition
Re_{min}	Reynolds number for zero-flow crossing
Re_{range}	Reynolds number range for laminar-turbulent transition
Re_t	Reynolds number limiting parameter for turbulent region
r	resistance coefficient
r_h	relative hydraulic roughness
r_{tf}	power transformation coefficient
s_k	sign parameter of port k
T	temperature
T_{ref}	reference temperature
t, t^*	time
t_0	initial time
v	enclosed fluid volume
v_0	initially enclosed fluid volume
x	displacement
x_{max}	maximal displacement value
γ_{oil}	thermal expansion factor of oil
γ	volume flow exponent
ζ	parameter of Roelands relation
η	viscosity of working fluid
η_{air}	viscosity of air
$\eta_{air,ref}$	reference viscosity of air
η_{oil}	viscosity of hydraulic oil
$\eta_{oil,ref}$	reference viscosity of hydraulic oil
η_{ref}	reference viscosity of fluid
θ	auxiliary function in equation for air viscosity
κ_{oil}	compressibility factor of oil
λ	friction factor
λ_l	friction factor, laminar region
λ_t	friction factor, turbulent region
μ_{oil}	relative mass part of hydraulic oil
μ_{air}	relative mass part of air
ξ	parameter of Roelands relation
ρ	density of working fluid
ρ_{air}	density of air
$\rho_{air,ref}$	reference density of air
ρ_{oil}	density of hydraulic oil
$\rho_{oil,ref}$	reference density of hydraulic oil
ρ_{ref}	reference density of fluid
ϕ_{air}	relative volume part of air
ϕ_{oil}	relative volume part of hydraulic oil
χ	auxiliary function in pipe friction factor equation
ψ	auxiliary function in Roelands relation